The role of the spatial dependence of the electron effective mass in forming the Wannier-Stark spectrum

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Corrigendum

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The author sincerely thanks Professor Yu V Kopaev for pointing out an important mistake made in the above paper. The model presented in the paper, which is based on the effective mass approximation, pretends to reveal the inherent energy spectrum in the Wannier–Stark problem for infinite superlattices. At the same time, as was shown in the paper itself, for a given particle's energy E the transfer matrix $\mathcal{Z}_{(-\infty,0)}(E)\mathcal{Z}_{(1,\infty)}(E)$ to describe the whole periodic structure is expressed in terms of the one-period transfer matrix Z at the energies $E \pm n\Delta$ where $n = 0, 1, \ldots$ This means that the effective mass of a particle should be taken into account at all these energies too. However, this task cannot be performed because a part of these points fall into the energy gaps where the notion of the effective mass has no physical sense. Thus, the effective mass approximation is a poor basis for solving the Wannier–Stark problem for superlattices. This is true for any value of the electric field strength.

It should be stressed here that the model presented in the paper [1] for solving this problem in the case of lattices is entirely applicable to superlattices. As regards the model considered here, its results can be applied to all physical problems where the same (by form) equations appear, provided that the equation of the model is treated beyond the effective mass approximation, as the Schrödinger equation for a particle with a spatially dependent mass (described by the periodic piecewise constant function).

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The role of the spatial dependence of the electron effective mass in forming the Wannier–Stark spectrum

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Abstract. We have shown that in the effective mass approach the spatial dependence of the effective mass significantly influences electron properties in the Wannier–Stark problem. That is, if the effective mass of an electron in the infinite periodic structure is uniform everywhere, then its inherent energy spectrum is continuous. But if the effective mass in the adjacent layers of the periodic structure is different, then the energy spectrum must be discrete and consist of so-called Wannier–Stark ladders. To calculate the Wannier–Stark spectrum and the corresponding wave-functions, we proposed a formalism based on the transfer-matrix method. The formalism is used to investigate anticrossing of the levels of the different Wannier–Stark ladders.

1. Introduction

Although the study of the electron motion in periodic structures in a constant uniform electrical field (the Wannier–Stark (WS) problem) has a long history, there remain moot points in its solution. The main question that we will dwell on is as follows. On the one hand, as was shown experimentally (see for example [1]), the energy spectrum of an electron in superlattices (SLs) consists of the so-called WS ladders predicted by Wannier [2] (this is also attested to by reliable numerical calculations (see for example [3])). On the other hand, the theoretical analyses based on the one-dimensional single-particle Schrödinger equation with both vector [4] and scalar [5–7] potentials show that, for infinite periodic structures with any bounded (within a period) potential, its spectrum must be continuous. And only for periodic structures of the δ-potentials (under certain conditions) [4] and of the δ'-potentials [8] should the energy spectrum be discrete. Since the latter two models cannot approximate real structures whose potentials have finite discontinuities, this means that there is no theoretical model which would be suitable for SLs and have a discrete spectrum as a solution.

Particular attention is merited by an approach [9, 10] which treats the same equation as in [4–7], but yields a discrete electron spectrum. The main feature of this method is the inclusion of only a finite number of minibands in the expansions in the Bloch functions. In our view, this is equivalent to the fact that the probability of finding the electron in the minibands ignored should be zero, and, hence, the whole energy region above the uppermost allowed miniband thereby transforms into a gap. Then, in the tilted-bands representation, this means that the electron motion in this approach is spatially localized. Therefore, this method cannot serve as a tool for revealing the genuine electron spectrum in the WS problem for a given periodic structure. Rather, it is a tool for studying the WS states after the discreteness of the spectrum for the structure has been established. In particular, making use of this approach to

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solve the WS problem for lattices is, strictly speaking, incorrect because the spectrum in this case is continuous irrespective of the electric field strength [7].

However, as will be seen below, the application of this method [9, 10] to SLs is well grounded. The point is that the Schrödinger equation, which has been exploited in theoretical analyses of the WS problem, beginning with Wannier's paper [2], is well suited to describe lattices but it is very crude for SLs. It is known [11, 12] that the simplest equation describing the motion of an electron in a SL is of the Sturm–Liouville type, but is not the Schrödinger equation. Now the electron (as a quasi-particle) has (effective) mass which can be changed over the structure. In this paper we show that it is the spatial dependence of the effective mass that plays a decisive role in forming the discrete spectrum for SLs. Thus, the effects of the electrical field on an electron in the lattices and SLs differ qualitatively (except in the special case of SLs with everywhere uniform effective mass). Therefore, in the WS problem we will further distinguish the lattice model whose solution is presented in [7] and the SL model (SLM) which is considered below (by means of this terminology, SLs with strongly uniform effective mass are described by the lattice model).

It is interesting to note that, in the numerical modelling of SLs (see for example [3]), the difference in the effective masses of the adjacent layers is taken into account. However, as we know, the fact that the electron spectrum in the problem can be qualitatively transformed, as one passes from the model with the everywhere constant electron mass to that with different masses in different layers, has not been discussed in the literature. It is generally agreed that the electron spectrum in the problem depends only on the potential form and the electric field strength.

2. The model

The model of SLs in the electrical field which will be considered here coincides with that presented in [7], except that the electron masses $m^*(x)$ in the out-of-barrier and barrier regions are different (the terminology and notation used here are adopted from [7]). In addition, the effective mass is assumed to be constant within the layers. This model can describe SLs formed from two alternating layers of different materials.

We recall that in the SLM the electron motion is governed by an equation of the Sturm–Liouville type [11, 12]. For each region where the effective mass is constant, it coincides with the Schrödinger equation—that is,

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + \frac{2m^*}{\hbar^2}\big(E - V(x)\big)\Psi = 0 \tag{1}$$

where E is the electron energy; the potential $V(x)$ is determined by the following expressions:

$$V(x) = \begin{cases} v(x) - n\Delta & \text{if } x \in (a_n, b_{n+1}) \\ -n\Delta & \text{if } x \in [b_n, a_n] \end{cases} \qquad (n = 0, \ldots, N-1)$$

where $b_n = nD$; $a_n = l + nD$ $(n = 0, \ldots, N)$; $\Delta = e\mathcal{E}D$; e is the (modulus of the) electron charge; l is the width of the out-of-barrier regions; D is the SL period; $\mathcal{E}$ is the electric field strength; and $v(x)$ is a bounded D-periodic function. The electron effective mass is such that $m^* = m$ for the barrier regions and $m^* = m_0$ for the out-of-barrier ones; here m_0 and m are constants. At the boundaries of adjacent regions, the function $\Psi'(x)/m^*(x)$ must be continuous.

The main goal of the paper is, as in [7], to find the stationary states $\Psi_{\mathcal{E}}(x; E)$ for an electron which correspond to the symmetry of the problem. We recall that the symmetry condition for

this problem may be written as (see [7])

$$\Psi_{\mathcal{E}}(x + D; E) = \text{constant} \times \Psi_{\mathcal{E}}(x; E + \Delta) \tag{2}$$

where the constant is a complex value. In the lattice model, such functions have been shown to exist, and their calculation is reduced to solving the functional equation for the coefficients of the general solution of equation (1) for the out-of-barrier region of the zero cell [7].

Let us write down the general solution of equation (1) for the out-of-barrier regions in the form

$$\Psi(x; E) = A_n^{(+)}(E) \exp[\mathrm{i}k_n(x - b_n)] + A_n^{(-)}(E) \exp[-\mathrm{i}k_n(x - b_n)] \tag{3}$$

where $k_n = \sqrt{2m_0(E + n\Delta)/\hbar^2}$; $n = 0, \ldots$. The coefficients in expression (3) for the wave-functions required must satisfy, as was shown in [7], the functional equation

$$\mathcal{A}_0(E) = C(E)Z(E)\mathcal{A}_0(E + \Delta) \tag{4}$$

where $C(E)$ is a complex function, and

$$Z = Y\Gamma = \begin{pmatrix} q & p \\ p^* & q^* \end{pmatrix} \qquad \mathcal{A}_n = \begin{pmatrix} A_n^{(+)} \\ A_n^{(-)} \end{pmatrix}. \tag{5}$$

Here Y is the transfer matrix (see [7]) describing the barrier region (on the assumption that there is no step on the right-hand side of the barrier), and $\alpha\Gamma$ is the matrix matching the solutions at the step. Also

$$\alpha(E) = \sqrt{k_1(E)/k_0(E)}.$$

It should be noted that the matching matrices in our model and the lattice model are the same. However, the transfer matrices Y describing the barrier are different. In particular, the transmission coefficient T for the rectangular barrier is determined, as follows from [13], by the expression (for the above-barrier case)

$$T = \left(1 + \theta^2 \sin^2(\varphi)\right)^{-1} \tag{6}$$

where

$$\begin{aligned} &\theta = \frac{1}{2}\left(\frac{\tilde{\kappa}_0}{\tilde{\kappa}} - \frac{\tilde{\kappa}}{\tilde{\kappa}_0}\right) && \varphi = \kappa d \\ &\tilde{\kappa}_0 = \kappa_0/m_0 && \tilde{\kappa} = \kappa/m \\ &k = \sqrt{2m(E - V_0)/\hbar^2}. \end{aligned} \tag{7}$$

Here, V_0 is the height of the rectangular barrier.

Recall that the solution of equation (4) depends fundamentally on the asymptotics of the function $T(E)$ [7]. From (7) it follows that, as $E \to \infty$,

$$\theta = \begin{cases} 0 & \text{for } m = m_0 \\ \theta_0 & \text{for } m \neq m_0 \end{cases} \qquad \text{where } \theta_0 = \left(\frac{m}{m_0}\right)^{1/2} - \left(\frac{m_0}{m}\right)^{1/2}.$$

Thus, unlike in the case for the lattice model where the transmission coefficient, with increasing energy, rapidly approaches unity (see also [7]), the function $T(E)$ in the SLM oscillates, at large energy, between unity and $(1 + \theta_0^2)^{-1}$. That is, the electron 'feels' the jumps of the kinetic energy at the layer boundaries at all values of E. As a result, the energy spectra in the lattice model and in the SLM, in the absence of the electrical field, are essentially different. That is, in the first case the energy gap approaches zero with increasing gap index, but in the second case, in contrast, it grows infinitely (see figure 1) (as in the model with the δ'-potential [8]). This leads to functional equation (4) having only a trivial solution, and, therefore, another approach is required in the SLM to search for the wave-functions obeying the symmetry condition of the problem.

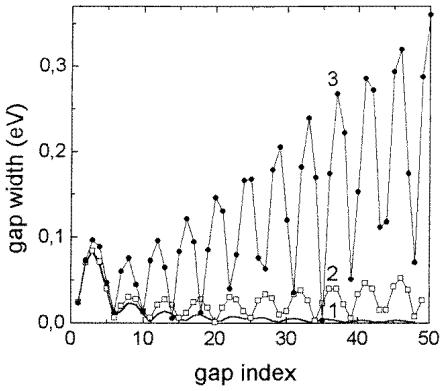

Figure 1. The gap width as a function of the gap index for three values of m: (1) $m = 0.067m_e$, (2) $m = 0.07m_e$, (3) $m = 0.093m_e$; $V_0 = 0.243$ eV, $d = 25$ Å, $l = 95$ Å, $m_0 = 0.067m_e$.

3. The Wannier–Stark spectrum

Because the gap width grows infinitely together with the gap index, the transmission coefficient $T_{(1,\infty)}$ for the semi-infinite structure positioned to the right of the zero cell must be zero *a priori* (recall that in the lattice model [7] $T_{(1,\infty)} \neq 0$ at any electron energy). Thus, the semi-infinite structures disposed to the left and to the right of the zero cell are both absolutely opaque to an electron with a variable effective mass; hence, its motion should be spatially localized. In this case the wave-function must tend to zero when the number of the cell, which is sufficiently far from the zero cell, grows.

To find the corresponding wave-functions, we proceed as follows. Recall that the solutions for the zeroth and Nth cell located to the right of the former are connected by the expression (see [7])

$$\tilde{\mathcal{A}}_N(E) = \mathcal{Z}^{-1}_{(1,N)}(E)\mathcal{A}_0(E) \tag{8}$$

where

$$\tilde{\mathcal{A}}_N(E) = \alpha_{(1,N)}(E)\mathcal{A}_N(E) \qquad \alpha_{(1,N)}(E) = \prod_{n=0}^{N-1}\alpha(E + n\Delta)$$

$$\mathcal{Z}_{(1,N)}(E) = Z(E)\cdots Z(E + (N-1)\Delta) \equiv \begin{pmatrix} Q_{(1,N)} & P_{(1,N)} \\ P^*_{(1,N)} & Q^*_{(1,N)} \end{pmatrix}$$

$$Q_{(1,N)} = \frac{1}{\sqrt{T_{(1,N)}}}\exp(-\mathrm{i}J_{(1,N)}) \qquad P_{(1,N)} = \sqrt{\frac{R_{(1,N)}}{T_{(1,N)}}}\exp\left[\mathrm{i}\left(\frac{\pi}{2} + F_{(1,N)}\right)\right].$$

Let us require that $\tilde{\mathcal{A}}_N(E) \to 0$ when $N \to \infty$. This occurs, as may be shown by using equation (8), when the general-solution coefficients for the zero cell are determined by the expression

$$\frac{A_0^{(-)}(E)}{A_0^{(+)}(E)} = \exp\left[-\mathrm{i}\left(J_{(1,\infty)} - F_{(1,\infty)} - \frac{\pi}{2}\right)\right] \tag{9}$$

where $J_{(1,\infty)}$ and $F_{(1,\infty)}$ are the parameters of the transfer matrix $\mathcal{Z}_{(1,\infty)}$. It should be noted here that this solution exists only when $T_{(1,\infty)} = 0$. This equality fails in the lattice model.

The limits $\lim_{n\to\infty} J_{(1,N)}$ and $\lim_{n\to\infty} F_{(1,N)}$ in (9) do not exist separately. But the limit $\lim_{n\to\infty}[J_{(1,N)} - F_{(1,N)}]$ does exist. This may be shown by making use of the scattering-matrix

formalism. The point is that the right-hand term in equation (9) is just the element $S_{21}^{(1,\infty)}$ of the scattering matrix corresponding to the transfer matrix $\mathcal{Z}_{(1,\infty)}$. The rapid convergence of $S_{21}^{(1,N)}$ with increasing N for this case is evident from the recurrence relation (see for example [14])

$$S_{21}^{(1,N+1)}(E) = S_{21}^{(1,N)}(E) + S_{22}^{(1,N)}(E)S_{11}^{(1,N+1)}(E)p^*(E + N\Delta).$$

That is, it takes place because the matrix elements $S_{11}^{(1,N)}$ and $S_{22}^{(1,N)}$ decrease exponentially when N increases, and the corresponding Nth cell is in the remote tilted energy gap. As we have shown, the energy gap grows infinitely with increasing index.

A similar requirement for the wave-function in the limit $x \to -\infty$ provides another expression for the vector $\mathcal{A}_0$:

$$\frac{A_0^{(-)}(E)}{A_0^{(+)}(E)} = \exp\left[\mathrm{i}\left(J_{(-\infty,0)} + F_{(-\infty,0)} - \frac{\pi}{2}\right)\right] \tag{10}$$

where $J_{(-\infty,0)}$ and $F_{(-\infty,0)}$ are the parameters of the transfer matrix $\mathcal{Z}_{(-\infty,0)}$ describing the semi-infinite structure positioned to the left of the zero cell. The right-hand term in (10) coincides with the element $S_{12}^{(-\infty,0)}$ of the scattering matrix that corresponds to the transfer matrix $\mathcal{Z}_{(-\infty,0)}$. So the existence of this limit is proved in the same manner as in the previous case. But now one has to rely on the recurrence relations for the scattering matrix $S_{(-N,0)}$.

The compatibility condition for expressions (9) and (10) provides the equation for the energy eigenvalues:

$$J_{(-\infty,0)} + F_{(-\infty,0)} + J_{(1,\infty)} - F_{(1,\infty)} = k\pi \tag{11}$$

where k is an odd number.

Note that equation (11) coincides in form with the perfect-transparency condition for two-barrier structures [15]. In this case the two-barrier structure is merely the whole infinite SL immersed in the electrical field, which is described by the matrix $\mathcal{Z}_{(-\infty,0)}\mathcal{Z}_{(1,\infty)}$. One can easily show that for the infinite structures the left-hand side of equation (11) is invariant when the electron energy varies by a quantity divisible by Δ. So this equation yields the WS spectrum, and the corresponding wave-functions satisfy symmetry condition (2). In this case the constant is equal to unity for the normalized functions.

In the framework of our approach we did not succeed in establishing the number of different WS ladders. However, according to the papers [9, 10], each WS ladder relates to some miniband. Thus, as the number of minibands is infinite, the same holds for the WS ladders. From which miniband some ladder originates may be judged from the localization character of the wave-function (near anticrossing, the situation is more complicated).

To calculate the eigenvalues, it is sufficient to investigate equation (11) over the interval $(0, \Delta]$, where they form, in accordance with the above, an infinite countable set. However, we should point out the fact that the phases in the left-hand term of equation (11), near the levels corresponding to the minibands in the high-energy region, change so quickly that these levels can be lost in the numerical estimation. Therefore, to search for these levels, it is desirable to investigate the energy interval in which the mid-point of the miniband considered is contained.

4. Wave-functions and the anticrossing of levels

Since the set of energy levels in the interval $(0, \Delta]$ is infinite, it would appear reasonable for the changes in the electric field strength or in any parameter of the SL to lead to crossing of the levels. But in the one-dimensional case, degeneracy of the energy levels is known to be impossible. Therefore, what actually happens is the so-called anticrossing of the levels [16–18].

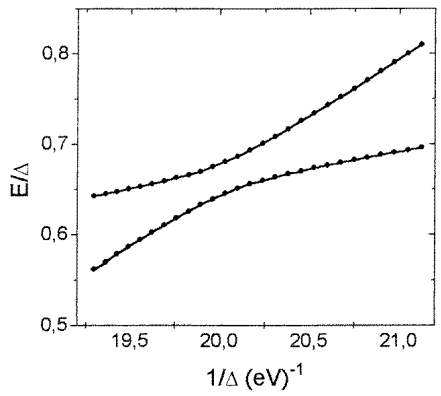

Figure 2. The anticrossing of the levels for the first two minibands; $m_0 = 0.067m_e$, $m = 0.093m_e$; the SL parameters are the same as for figure 1.

Interest in this has been increasing due to the Zener tunnelling (ZT) which occurs just at the anticrossings [16–18].

To demonstrate the efficiency of our approach for the numerical calculations of the spectrum and wave-functions, and in analysing the anticrossings, we have studied the SL using the following parameters (see [3]): $v(x) = V_0 = 0.243$ eV, $d = 25$ Å, $l = 95$ Å, $m_0 = 0.067m_e$, $m = 0.092m_e$; m_e is the free-electron mass. Figure 2 illustrates the anticrossing of the levels related to the first two minibands. The value of Δ varies over the interval $[0.047, 0.052]$ eV. Figures 3(a)–3(c) give the wave-functions for the three values of the energy for the upper curve in figure 2. It is seen how the wave-function related to the first miniband (figure 3(a)) transforms, when one follows along the curve from left to right, into that related to the second miniband (figure 3(c)). (For the lower curve in this case, the state inherent to the second miniband transforms into the state inherent to the first one.) At the anticrossing point where the two levels are nearest to each other, the contributions from the two minibands are approximately equal (figure 3(b)). In this state the probabilities of the electron being in the spatial SL regions corresponding to the first and second minibands are practically equal. In the stationary case it is this point which attests to the efficiency of the ZT.

Particular attention should be given to the notion of ZT. Strictly speaking, it makes sense to consider this effect in the time-domain analysis—that is, as a dynamic process. In the vector-potential representation, the ZT appears as the transitions of a Bloch electron from the lower minibands to the upper ones. When the scalar potential is used, it appears as the transitions between corresponding spatial regions. This correspondence arises due to the symmetry of the problem (see condition (2)). It should be noted that the wave-functions must be current carrying, in order to describe the tunnelling process. But the stationary states in the SLM are localized—that is, with zero current. Therefore, they cannot describe the tunnelling process. Nevertheless, the corresponding wave-functions provide some information about the efficiency of the ZT for an electron with a given value of E. Note that the localization character of the wave-functions changes sharply at the anticrossing points. Far from these points, the electron is localized in one miniband. That is, in this case ZT is practically absent. But near the anticrossing points the electron may occupy the spatial regions corresponding to two or more minibands; that is, the so-called mixing of one-miniband states occurs. This attests to the high efficiency of ZT for such an electron.

With decreasing Δ the anticrossing regions for which the ZT can be observed become

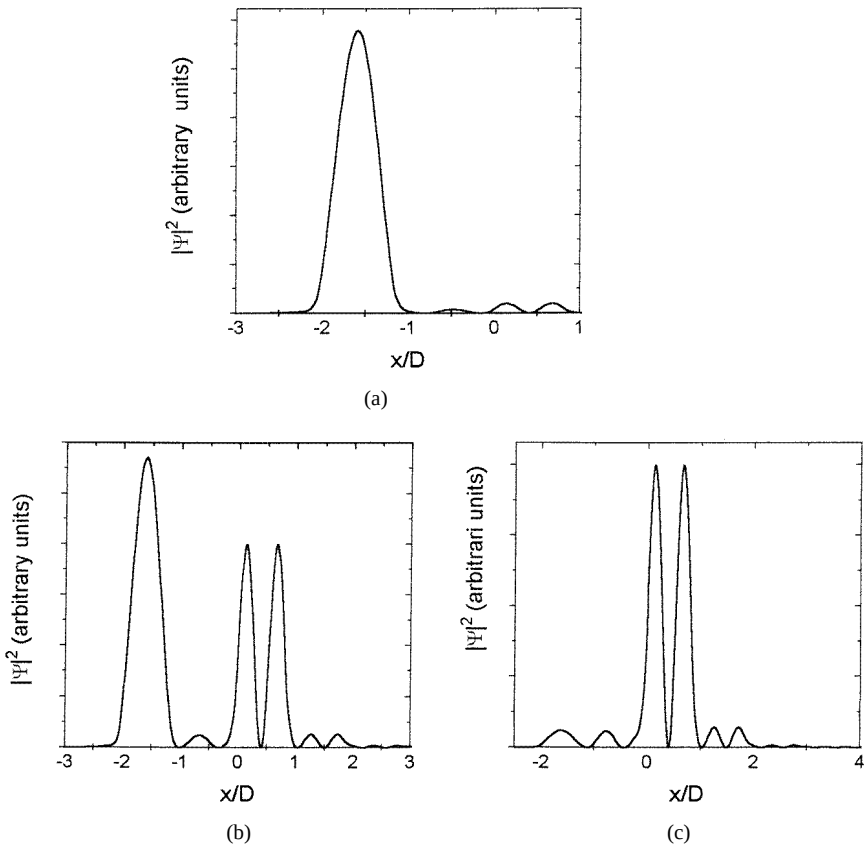

Figure 3. (a) The wave-function at $E = 2.640\,407\,649\,46\,\Delta$ (the first miniband) for the upper curve in figure 2 at $\Delta = 0.052$ eV; the SL parameters are the same as for figure 1. (b) The wave-function at $E = 2.677\,749\,4743\,\Delta$ (near the anticrossing of the levels corresponding to the first two minibands) for the upper curve in figure 2 at $\Delta = 0.050\,02$ eV; the SL parameters are the same as for figure 1. (c) The wave-function at $E = 2.773\,119\,8675\,\Delta$ (the second miniband) for the upper curve in figure 2 at $\Delta = 0.048$ eV; the SL parameters are the same as for figure 1.

narrow, and the requirements imposed on the accuracy of the estimation of the energy levels become more rigid. Figure 4 shows the wave-function at $\Delta = 5.783\,356\,93\hbar^2/(m_e D^2)$. It is seen that the localization character of the wave-function in this case is unusual. As in the previous case, the electron near the anticrossing may be found, with approximately equal probability, in either of the two spatial regions which correspond to the first two minibands. But now there is a relatively wide range (corresponding to the tilted gap between the first and second minibands) over which the probability of finding the electron is about nil. The anticrossing domain at a given Δ is narrow to the extent that any change in Δ in the ninth significant figure causes suppression of the ZT.

The analysis of the model shows that the mixing effect and, as a consequence, the ZT depend fundamentally on the parameter δm, where $\delta m = m - m_0$. As is seen from figure 1, at $\delta m \neq 0$ the envelope of the gap width as a function of the gap index has a minimum. The smaller $|\delta m|$, the bigger the gap index for which the minimum occurs. As a consequence, the number of minibands separated by fairly small gaps increases. This means that the mixing effect is strengthened, and hence stationary states formed by more than two minibands appear.

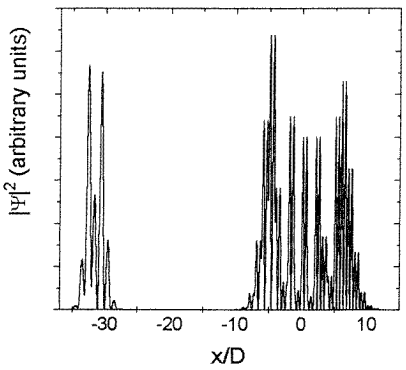

Figure 4. The wave-function at $E = 42.602\,065\,509\,074\,31\,\Delta$ for $\Delta = 5.783\,356\,93\,\bar{h}^2/(m_e D^2)$ (the case of a weak electrical field); the SL parameters are the same as for figure 1.

When $\delta m \to 0$, the localization region for an electron expands to infinity. But if $|\delta m|$ is infinitely small and not equal to zero, the stationary states remain, evidently, localized; only for the exact equality, $\delta m = 0$, do they become delocalized. In this case we come to the lattice model [7].

5. Conclusions

The main result of this paper and our previous research [7] is that the energy spectra for an electron in periodic structures, lattices, and superlattices, in constant uniform electrical fields, differ qualitatively. For comparison, let us review the most important properties of an electron in the two models.

5.1. The model for lattices and for SLs with everywhere uniform effective mass

The transmission coefficient for the semi-infinite structure positioned in the classically accessible region is not equal to zero at any value of the electron energy and electric field strength. The ZT between the lowest pair of minibands can be very weak. But for those with large indices it is always significant. As a result, the stationary states for an electron in infinite structures have, as a rule, (i) a more or less distinct region where the probability of finding the electron is rather large and (ii) a 'tail' for $x \to \infty$ where the probability diminishes as a power law [7]. Thus, in this model, the eigenfunctions are delocalized in the direction of the OX axis, and the energy spectrum for an electron is continuous. Note that the stationary wave-functions for an electron in infinite structures do not satisfy the symmetry condition (2) (functions obeying this condition are unbounded at minus infinity [7]). There are only damped Bloch oscillations in these periodic structures. They can be observed only for the lowest minibands, because in this case the damping is sufficiently weak. The general tendency is such that the bigger the miniband index, the stronger the damping.

5.2. The model for superlattices with different effective masses in adjacent layers

In this model the semi-infinite structures situated in the classically inaccessible and in the classically allowed regions are completely opaque to an electron. As a result, the stationary states for an electron in infinite superlattices are spatially localized, and its energy spectrum is

a discrete one of the WS type. The ZT for an electron in a stationary state is important only at the anticrossings. It is appreciable here only for the minibands in the lower part of the energy scale. This is because the gaps in the upper part of the energy scale enlarge with increase of their indices, and, as a consequence, the width of the anticrossing regions tends to zero.

Note that the multi-band approach developed in [9, 10] is appropriate just for this model. But it should be noted that the model is very sensitive to changes in the parameter δm. The basic reason for this is that, for the energy gap W_N with index N, the inequality

$$\lim_{\delta m \to 0} \lim_{N \to \infty} W_N \neq \lim_{N \to \infty} \lim_{\delta m \to 0} W_N .$$

holds.

In contrast to the case for the lattice model, the wave-functions for an electron in infinite superlattices of general form obey symmetry condition (2) (the constant is equal to unity for normalized functions). The electron, in this case, may undergo undamped Bloch oscillations when the electrical field does not correspond to an anticrossing point. Otherwise, the electron motion covers more than one miniband and therefore it should have a more complicated character. The model presented opens up, in our view, new possibilities for detailed study of the electron dynamics in superlattices subjected to electrical fields. In particular, it would be interesting to investigate in detail the period of the Bloch oscillations, which is believed to depend only on Δ.

We hope that both of the models will help to clarify the moot points (in particular, the question regarding the inherent energy spectrum) in the WS problem. The intriguing point is that all of the parties participating in the dispute on this question are right to some extent—that is, those who claim that the spectrum should be continuous and those who advocate a discrete spectrum. The salient point is which periodic structure is investigated. If it is a lattice or superlattice with everywhere uniform effective mass then the spectrum must be continuous. If it is a superlattice with non-uniform effective mass then the spectrum must be discrete, as was predicted by Wannier [2] (but in this case the study ought to have been performed beyond the Schrödinger equation).

Of course, the SLM should be considered only as one step in constructing a correct theory for the WS problem for superlattices. As is seen from this paper, the solution of the problem depends fundamentally on the asymptotics of the transfer matrix in the high-energy region. But this is the very region where the effective-mass approach, in the given form, becomes crude. The more correct equations for the wave-function envelope have to be the basis of future analyses of the WS problem for superlattices.

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